

REDUNDANCY ANALYSIS OF EXISTING TRUSS BRIDGES: “A SYSTEM RELIABILITY-BASED APPROACH”

Tarek N Kudsi* and Chung C Fu†

* Senior Structural Engineer, Ph.D.

Dar Al Handasah Consultants (Shair & Partners)

Dar Al Handasah Building, Verdun Street, P.O.Box 7159

Beirut, Lebanon

Email: Tarek.Kudsi@darbeirut.com, Phone: 961-1-790002/3/4

† Associate Professor, Ph.D., P.E.

University of Maryland at College Park

College Park, MD 20742, USA

Email: ccfu@eng.umd.edu, Phone: 1-301-405-2011

Key Words: Reliability, Redundancy Analysis, Systems Analysis, Truss Bridges,

Abstract. *The main objective of this study is to develop a new methodology for redundancy analysis of structural systems in general and truss bridges in particular.*

The proposed methodology depicts the required steps to build a block diagram, composed of members in series and parallel configuration, representing the structural system or the truss bridge. The proposed block diagram accounts for the system's degree of redundancy and the possible amount of redundant members' combinations, in a parallel configuration, to be laid in series with the non-redundant members in the system. Multiple failure modes, related to the system's components, are integrated in the structural system in a series system layout. General equations are then derived for the pre-failure and post-failure phases of the system's reliability index and probability of failure. The post-failure phase is defined as the phase when a redundant member fails without causing the collapse of the system. The proposed methodology is then applied to a truss bridge. Ten cases are presented, representing the system's health and its updated block diagram following modification of four lower chord members separately and the complete failure of six redundant members separately. The proposed methodology is applied on each case in order to categorize the truss bridge as a very redundant, redundant, or a non-redundant system.

INTRODUCTION

Steel railroad truss bridges have been in use since 1779 and have proven to be strong and durable under high live load impact conditions. Their adaptation to the highway environment also has proven to be highly successful and useful for intermediate span lengths (i.e., 36 m to 105 m).

Over the past three decades, the demand for truss bridges has decreased due to a variety of reasons, many of which have little to do with their original advantages of strength and economy. Some of the reasons are the availability and subsequent improvements of alternate construction types, such as steel plate girders and pre-stressed concrete.

Throughout the years, researchers have developed many methodologies to truly analyze the behavior of truss bridges and depict their true redundancy in the case of collapse, based on deterministic and probabilistic techniques. The latest NCHRP study dealing with the issue of redundancy was in report number 406⁷ (NCHRP 1998), where redundancy is defined as the capability of the bridge to continue to carry loads following the failure (or damage) of one or more of its members. The analysis presented in the NCHRP report number 406, is based on probabilistic and reliability techniques

In this study, a new methodology is developed for redundancy analysis of structural systems in general and truss bridges in particular based on reliability techniques.

BASIS OF RELIABILITY ENGINEERING

The reliability of engineering systems can be presented as a problem of supply versus demand. *Traditionally, the reliability of engineering systems is achieved through the use of safety factors and adopts conservative assumptions in the process of design*². In addition, the existing supply and the required demand may be modeled as random variables. Examples of random variables are: the cross section of a beam, the moment of inertia, the flange depth and thickness, the material strength, the elastic properties, and the applied loads on the structure such as vehicle loading, wind loading, or earthquake loading. In order to better understand the mentioned random variables, samples should be gathered and analyzed so as to get the mean and coefficient of variation (or standard deviation) in order to incorporate them into their respective distribution functions.

Many researchers including Nowak¹⁰, Frangopol⁴, Ghosn and Moses⁷, have gathered information on family of bridges, and their findings have laid a foundation for reliability-based design of bridge structural systems.

Probability Of Failure

Failure occurs when the load effect (L) exceeds the resistance (R) of the structure, and can be derived by considering the probability density functions of R and L, along with their associated random variables. The main goal for the safety of the structure is to guarantee an $R > L$ scenario throughout the design life of the structure. Assuming that R and L are normal, statistically independent, and positive random variables, it can be stated that⁵:

$$P_f = \int_{-\infty}^{\infty} \int_{-\infty}^{L_j \geq R_i} f_R(R_i) f_L(L_j) dR_i dL_j \quad (1)$$

$f_R(R_i)$ is the probability density function for the structural resistance, and $f_L(L_j)$ is the probability density function of the external loading.

The reliability index is defined as:

$$\mathbf{b} = -\Phi^{-1}(P_f) \quad (2)$$

System Reliability

A system is categorized as a series or a parallel system as follows:

1. For series system the probability of failure limits are (Figure 1):

$$\max_i^n (P_{f_i}) \leq P_f \leq 1 - \prod_{i=1}^n (1 - P_{f_i}) \quad (3)$$

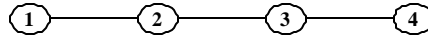


Figure 1: SERIES SYSTEMS

2. For parallel systems, the probability of failure limits are (Figure 2):

$$\prod_{i=1}^n P_{f_i} \leq P_f \leq \min_{i=1}^n P_{f_i} \quad (4)$$

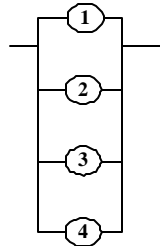


Figure 2: PARALLEL SYSTEMS

REDUNDANCY ANALYSIS OF TRUSS BRIDGES; A NEW APPROACH

Modeling a bridge as a collection of structural elements in series and parallel in a structural system is very essential for evaluation of its true redundancy and reliability.

In order to build the structural system, the following need to be carried out:

1. Analyze the bridge structural system.
2. Fail each element separately, and re-analyze the bridge. Reliability indices are generated.

3. Incorporate the elements in the system. If the element is redundant, then it is in parallel with the rest of the system. If the element is non-redundant, then it is in series with the rest of the system.
4. Define the degree of redundancy of the system (D.O.R), k , based on the number of members, which can be cut simultaneously without resulting in the failure of the system, thus $k = 1, \dots, s$.
5. Number the non-redundant members from $i = 1, 2, \dots, n$.
6. Number the redundant members from $j = n + 1, n + 2, \dots, n + m$.
7. Build a block diagram for the truss bridge system according to steps 1 through 9, for the pre-failure phase, and also for the post-failure phase.

System Analysis In The Pre-Failure Phase

The following presentation is for redundant truss bridges systems in the pre-failure phase. The pre-failure phase is defined as the intact truss system, prior to any redundant member failure or modification.

Example system, as illustrated in Figure 3: A truss bridge composed of ten elements. If the degree of redundancy is $k = 2$, this implies that failure of any two redundant components will not cause the bridge to fail. The number of non-redundant members equals to five, $i = 1, 2, 3, 4, 5$, the number of redundant members equal to five, $j = 6, 7, 8, 9, 10$. Assuming that the components are fully correlated, $\rho = 1$, the probability of failure of the pre-failure phase is:

$$\begin{aligned}
 P_{f_{system_pre_failure}} = \max_i & (P_{f_i}, \min(P_{f_6}, P_{f_7}, P_{f_8}), \min(P_{f_6}, P_{f_7}, P_{f_9}), \\
 & , \min(P_{f_6}, P_{f_7}, P_{f_{10}}), \min(P_{f_6}, P_{f_8}, P_{f_9}), \min(P_{f_6}, P_{f_8}, P_{f_{10}}), \\
 & , \min(P_{f_6}, P_{f_9}, P_{f_{10}}), \min(P_{f_7}, P_{f_8}, P_{f_9}), \dots \\
 & , \min(P_{f_7}, P_{f_8}, P_{f_{10}}), \min(P_{f_8}, P_{f_9}, P_{f_{10}}))
 \end{aligned} \quad (5)$$

The above example system presented can be generalized for k degrees of freedom and m numbers of redundant members in a system. The probability of failure of the system in the pre-failure phase, $P_{f_{system_pre_failure}}$ can be presented as follows:

$$P_{f_{system_pre_failure}} = \max_{i,j} \left(P_{f_i}, P(F_j) \right) \quad (6)$$

where P_{f_i} = probability of failure of the non-redundant members, $P(F_j)$ = the minimum probability of the occurrence of the failure event, F , of each possible arrangement of the redundant members, according to the system degree of redundancy, k , for fully correlated redundant members, $\rho = 1$. In order to identify the amount of failure event arrangements, let $r = k + 1$ be the combination of the failure events of the redundant elements from a set of m amount of redundant elements, thus the amount of combination can be found as follows:

$$C_{r|m} = \frac{m!}{r!(m-r)!} \quad (7)$$

Let $C_j = F_j$ = the series of the multiple failure events arrangements of the redundant members in the system. For a degree of redundancy $k = 2$, $r = k + 1 = 3$, and $j = n + 1, \dots, n + m$ is the amount of redundant members, the following series represent the possible arrangements of the failure events of redundant members:

$$\begin{aligned}
 F_j^{n+m} = & F_{n+1} \cap F_{n+2} \cap F_{n+3}, F_{n+1} \cap F_{n+2} \cap F_{n+4}, \dots, F_{n+1} \cap F_{n+2} \cap F_{n+m}, \\
 & F_{n+2} \cap F_{n+3} \cap F_{n+4}, F_{n+2} \cap F_{n+3} \cap F_{n+5}, \dots, F_{n+2} \cap F_{n+3} \cap F_{n+m}, \\
 & \dots, F_{n+m-2} \cap F_{n+m-1} \cap F_{n+m}
 \end{aligned} \tag{8}$$

Similar equations such as equation (8) can be generated for $k = 1, \dots, s$, where $s < m$. The above explanation of $P(F_j)$ is only applicable for fully correlated members, $\rho = 1$. When $P(F_j)$ is incorporated in equation (8), the maximum of the minimum of each combination is compared with all the probability of failure of the non-redundant members, and the maximum probability of failure would be the probability of failure for the pre-failure phase of the system $P_{f_{system_pre_failure}}$. When the redundant members are not correlated, $\rho = 0$, the component $P(F_j)$ = the product of the probability of the occurrence of the failure event, F_j , of each possible arrangement of the redundant members, according to the system degree of redundancy, k .

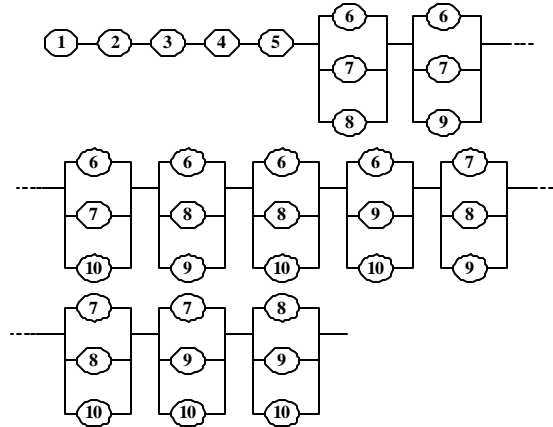


Figure 3: Pre-failure phase representation of the system

System Analysis In The Post-Failure Phase

The following example presents the probability of failure for the post-failure phase of any redundant system. The post-failure phase is defined as the phase when the truss system loses a redundant member or members. Let $l = 1, \dots, v$ represents the member or members failed in the system.

Example system, as illustrated in Figure 4: for this system, consider the failure of members 6 and 7. The degree of redundancy $k = 2$, $l = 6, 7$, this implies that the two redundant members 6 and 7

have failed. From Figure 4, the combination of 8-9, 8-10, 9-10, do repeat, for the calculation of the probability of failure of the system, only once, the combination will be counted. The probability of failure for the post-failure phase of the system is:

$$P_{f_{system}} = \max_i (P_{f_i}, P_{f_{j=n+1}}^{j=n+m}, \min(P_{f_8}, P_{f_9}), \min(P_{f_8}, P_{f_{10}}), \min(P_{f_9}, P_{f_{10}}), \min(P_{f_8}, P_{f_9}, P_{f_{10}})) \quad (9)$$

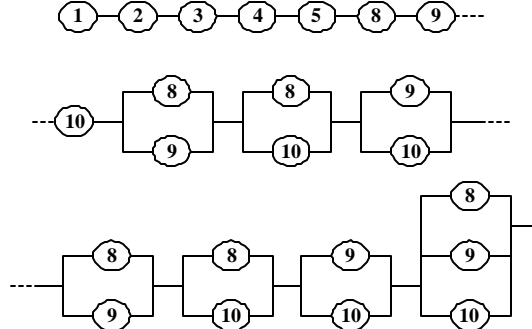


Figure 4: Post-failure phase representation of the system

The generalized probability of failure in the post-failure phase for a system of fully correlated components is:

$$P_{f_{system_post_failure}} = \max_i (P_{f_i}, P_{f_{j=n+1}}^{j=n+m}) \quad (10)$$

Reliability Measures For Redundant Truss Systems

The following equations can be derived for the pre-failure and post-failure phases from equations (6), and (10):

The probability of success of the system in the pre-failure phase is:

$$P_{s_{system_pre_failure}} = 1 - \max_{i,j} (P_{f_i}, P_{f_j}) \quad (11)$$

The probability of success of the system in the post-failure phase is:

$$P_{s_{system_post_failure}} = 1 - \max_i (P_{f_i}, P_{f_{j=n+1}}^{j=n+m}) \quad (12)$$

The reliability index of the system in the pre-failure phase is:

$$\mathbf{b}_{system_pre_failure} = -\Phi^{-1} \left(\max_{i,j} (P_{f_i}, P_{f_j}) \right) \quad (13)$$

The reliability index of the system in the post-failure phase is:

$$\mathbf{b}_{system_post_failure} = -\Phi^{-1} \left(\max_i (P_{f_i}, P_{f_{j=n+1}}^{j=n+m}) \right) \quad (14)$$

Reliability Limits for Redundant Truss bridges

The following limitations will depict if the system is redundant, in the pre-failure and post-failure phases, with respect to the pre-set target reliability index:

$$\text{If: } \mathbf{b}_{pre_failure} > \mathbf{b}_{post_failure} > \mathbf{b}_{target} \quad (15)$$

The component, the subsystem, or the global system, is highly redundant.

$$\text{If: } \mathbf{b}_{pre_failure} > \mathbf{b}_{post_failure} = \mathbf{b}_{target} \quad (16)$$

The component, the subsystem, or the global system, is redundant.

$$\text{If: } \mathbf{b}_{pre_failure} > \mathbf{b}_{post_failure} < \mathbf{b}_{target} \quad (17)$$

The component, the subsystem, or the global system, is non-redundant.

APPLICATION OF THE PROPOSED METHOD ON A TRUSS BRIDGE

The U.S Bridge Corporation Bridge 3000 is an example of a highway truss, which is prefabricated in standard segments and requires minimal field operation to construct. Reports written by Fu and Schelling, 1994 included the analysis procedures used in the design of the U.S Bridge model 3000 series. In this report both the traditional 2-D truss analysis and the 3-D finite element method are included. They provide the ability to determine both the primary stresses, in order to design the plane truss, and the secondary stresses throughout the truss-floor system, in order to determine levels of redundancy.

As a result, the model 3000 incorporates various levels of redundancy, such as:

1. Separate dual lower chord channel members, which provide a full HS-20 live load capacity if one chord is cut;
2. Alternate load paths as afforded by the interconnected trusses, and floor-beam and stringer system;
3. The use of A588 weathering or galvanized steel to increase resistance to corrosion and loss of section.

In 2000, Fu⁶ proposed a new improvement to the U.S Bridge Corporation Bridge 3000, by changing four diagonal members from the normal wide flanges to dual channels systems (members 8, 22, 41, and 55 in Figure 5). For this example, the gussets connecting the members were relied upon to redistribute the load and moment generated from the failure or modification of a member.

In this study, the redundancy analysis of the USB Truss Bridge 3000 is carried out and the results are shown in Table 1, based on the previously proposed methodology. The truss system is modeled as a block diagram composed of members in series and parallel configuration.

A detailed structural evaluation of the truss bridge is performed using a finite element software package, ANSYS 5.6. Also the target reliability index was taken to be $\mathbf{b}_{target} = 2.5$, and the reliability index in the pre-failure phase for the structural system is found to be $\mathbf{b}_{system_pre_failure} = 4.6$.

In order to build the truss bridge system, members are identified as redundant or non-redundant members. Three subsystems are identified and modeled as system in series:

1. Subsystem 1 refers to the left truss;
2. Subsystem 2 refers to the right truss;
3. Subsystem 3 refers to the floor beam and stringers system.

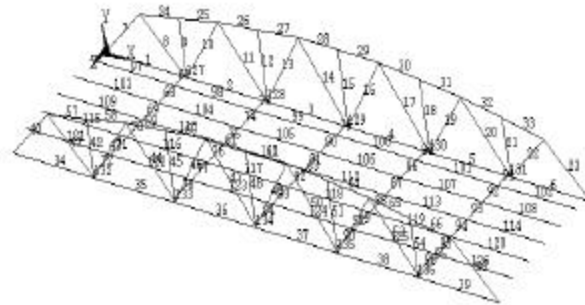


Figure 5: USB Bridge 3000 Isometric View with Element Numbering

Table 1: Summary of Results

Section	$b_{system_post_failure}$	Comments
Modification of Element 1	4.6	Very Redundant
Modification of Element 2	4.5	Very Redundant
Modification of Element 3	4.7	Very Redundant
Modification of Element 8	4.7	Very Redundant
Loss of Element 9	4.6	Very Redundant
Loss of Element 10	-0.9	Non-Redundant
Loss of Element 11	-1.5	Non-Redundant
Loss of Element 12	4.6	Very Redundant
Loss of Element 13	2.8	Very Redundant
Loss of Element 14	0.9	Non-Redundant

CONCLUSION AND FUTURE RESEARCH

The presented methodology is an efficient tool for a realistic representation of a structural system with redundant members by using a block diagram composed of members in series and parallel configuration. This methodology may eventually become a reliable tool to set new safety factors for redundant structural systems and truss bridges in future design codes, based on the calculated structural system reliability index. It can also be used in calibrating the strength of existing structural systems or truss bridges that experience a loss of a redundant member, in order to attain a target operational system reliability index. The same terminology can also be used to build a fault tree capable of accounting for the possible amount of redundant members in a structural system.

REFERENCES

1. AASHTO LRFD Manual, "AASHTO LRFD Bridge Design Specifications," Published by the American Association of State Highway and Transportation Officials (2000).
2. Ang, A.H., and Tang, W.H., "Probability Concepts in Engineering Planning and Design," Vol II, John Wiley and Sons, New York (1984).
3. Ayyub, B.M, and McCuen R.H., "Probability, Statistics, & Reliability for Engineers" CRC Press, Florida (1997).
4. Frangopol, D.M., and Nakib, R., " Redundancy in Highway Bridges." Engineering Journal, AISC, 28(1), pp. 45-50, (1991).
5. Freudenthal, A. M., Garrelts, J. M., and Shinozuka, M., "The Analysis of Structural Safety," Journal of Structural Division, Proc. ASCE, 92(ST1), pp. 267-325, (1966).
6. Fu, C. C., "Report on the Determination of Redundancy of the U.S. Bridge Corporation Bridge 3000, " submitted to Ohio Bridge, Cambridge Ohio, (2000).
7. Ghosn, M., and Moses, F., " Redundancy in Highway Bridges." NCHRP report 406, Department of Civil Engineering, The City College of The City University of New York, New York, NY, (1998).
8. Kudsi, T.N., "Redundancy Analysis of Existing Truss Bridges: A System Reliability-Based Approach," in partial fulfillment of the requirement for the degree of Doctor of Philosophy, Department of Civil and Environmental Engineering, University of Maryland, College Park, MD 20742, (2001).
9. Melchers, R.E., "Structural Reliability: Analysis and Predictions," Ellis Horwood Ltd., Chichester, (1987).
10. Nowak, A.S., "Calibration of LRFD Bridge Design Code, " NCHRP report 368, Department of Civil and Environmental Engineering, University of Michigan, Ann Arbor, Michigan, (1999).
11. Thoft-Christensen, P., and Murotsu, Y., "Application of structural system reliability theory," Springer-Verlag, Berlin, (1986).